

WEIGHTED INTERPOLATION FROM CERTAIN SINGULAR AFFINE HYPERSURFACES

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ABSTRACT. We prove that square integrable holomorphic functions (with respect to a plurisubharmonic weight) can be extended in a square integrable manner from certain singular hypersurfaces (which include uniformly flat, normal crossing divisors) to entire functions in affine space. This gives evidence to a conjecture [PV-2014] regarding the positivity of the curvature of the weight under consideration.

1. INTRODUCTION

Given a countable set of points $\Gamma \subset \mathbb{C}$, one can ask whether it is a set of interpolation for the Bargmann Fock space, i.e., whether for all collections of complex numbers $\{f_\gamma ; \gamma \in \Gamma\}$ such that

$$\sum_{\gamma} |f_\gamma|^2 e^{-|\gamma|^2} < \infty$$

one can find a holomorphic function F such that

$$F(\gamma) = f_\gamma \text{ for all } \gamma \in \Gamma \quad \text{and} \quad \int_{\mathbb{C}} |F|^2 e^{-|z|^2} d\mu_{Leb} < \infty.$$

More generally, one may ask the same question with the weight $e^{-|z|^2}$ replaced by $e^{-\phi}$ for some locally integrable function ϕ . If we impose that ϕ is \mathcal{C}^2 -smooth and satisfies $C^{-1} \leq \Delta\phi \leq C$, then a result due to Seip in the classical Bargmann-Fock setting, and to a number of authors in general, states that Γ is a set of interpolation for the generalized Bargmann-Fock space if and only if

- (1) Γ is uniformly separated with respect to the Euclidean metric, and
- (2) the so-called upper density

$$D_\phi^+(\Gamma) := \limsup_{r \rightarrow \infty} \sup_{z \in \mathbb{C}} \frac{\#(\Gamma \cap D(z, r))}{\int_{D(z, r)} \Delta\phi d\mu_{Leb}}$$

is strictly less than 1.

(See [S-2004] and the references therein).

Higher dimensional generalisations of this problem are of active interest and current research. [OSV-2006, BT-1982, B-1983, D-1980, PV-2014]. In higher dimensions, the problem is characterize those hypersurfaces $W \subset \mathbb{C}^n$ with the property that given any holomorphic function $f : W \rightarrow \mathbb{C}$ satisfying $\int_W |f|^2 e^{-\phi} < \infty$, there is a holomorphic function F that extends f and satisfies $\int_{\mathbb{C}^n} |F|^2 e^{-\phi} < \infty$. If we assume that ϕ is smooth and satisfies

$$(1.1) \quad C^{-1} \sqrt{-1} \partial \bar{\partial} |z|^2 \leq \sqrt{-1} \partial \bar{\partial} \phi \leq C \sqrt{-1} \partial \bar{\partial} |z|^2,$$

then sufficient conditions exist: In [OSV-2006] sufficient conditions were provided to solve this problem for smooth W . Akin to “uniform separation”, a geometric notion called “uniform flatness” was defined, as was a corresponding generalisation of the notion of “upper density”. It was then proved that if W is uniformly flat and has density strictly less than 1, then W is an interpolation

hypersurface. The reason for the lower bounds on $\partial\bar{\partial}\phi$ is that the desired interpolating function was constructed by “patching up” local extensions using the Hörmander theorem (which in turn requires strict positivity of the curvatures involved). The upper bound on $\partial\bar{\partial}\phi$ is related to the approach taken in [OSV-2006] to extend the data from W to a small neighbourhood. In [PV-2014], the same result for smooth hypersurfaces W was established for any plurisubharmonic weight ϕ . The main tool used was an Ohsawa-Takegoshi type extension theorem (theorem 3.2). As far as the necessity, little is known. The necessity of the density condition is wide open, but it was also shown in [PV-2014] that uniform flatness is not a necessary condition as soon as $n \geq 2$.

In [PV-2014] the problem of weighted interpolation for singular W was also studied. A corresponding notion of uniform flatness was defined for singular varieties and was proven to be one of the sufficient conditions (the other one being upper density less than 1) required for the solution of the interpolation problem provided the weight ϕ satisfies (1.1). Once again the Hörmander theorem was used in the singular case to patch up local extensions. It was conjectured that the condition on the weight may be weakened. The purpose of this paper is to provide evidence for this conjecture by weakening the strictness of plurisubharmonicity of ϕ for certain singular hypersurfaces (that include the case of “uniformly flat” simple normal crossing divisors). The strategy of proof is to use the Ohsawa-Takegoshi type theorem 3.2 (as in [PV-2014] for the smooth case) to solve the problem of weighted interpolation.

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2. STATEMENTS OF RESULTS

Let $\omega_0 = \frac{\sqrt{-1}}{2} \sum dz^i \wedge d\bar{z}^i$ be the Euclidean Kähler form on \mathbb{C}^n . We recall a few definitions before stating our main theorem.

Firstly, we define uniform flatness [OSV-2006] for smooth hypersurfaces in \mathbb{C}^n .

Definition 2.1. A smooth hypersurface $W \subset \mathbb{C}^n$ is said to be uniformly flat if there exists a tubular neighbourhood of W of radius ϵ_0 where ϵ_0 is a positive constant.

Next we recall the definition of the same concept in the special case of singular hypersurfaces of the type $W = T^{-1}(0)$ where $T = T_1 T_2 \dots T_k$ is an entire function such that $W_i = T_i^{-1}(0)$ is a smooth, uniformly flat hypersurface.

Definition 2.2. A singular hypersurface $W = \bigcup_{i=1}^k W_i$ where W_i are smooth hypersurfaces is said to be uniformly flat if the W_i are uniformly flat, and there is a positive constant ϵ_0 (which is the radius of the uniform tubular neighbourhood of the W_k) so that the angles of intersection $W_k \cap W_l$ lie in $[\epsilon_0, \pi - \epsilon_0]$.

Finally, we define the concept of density [OSV-2006].

Definition 2.3. Let W be a hypersurface. The $(1,1)$ -form $\Upsilon_r^W = [W] * \frac{1_{B(0,r)}}{\text{vol}(B(0,r))}$ is called the density tensor of W . If $\phi : \mathbb{C}^n \rightarrow \mathbb{R}$ is a plurisubharmonic function, then with $\phi_r(z) = \int_{B(z,r)} \phi(\zeta) \omega_0^n(\zeta)$, the number

$$(2.1) \quad D_\phi^+(W) = \inf \left\{ \alpha \geq 0 ; \sqrt{-1} \partial\bar{\partial}\phi_r - \frac{1}{\alpha} \Upsilon_r^W \geq 0 \text{ for all } r \gg 0 \right\}$$

is called the upper density of W .

Remark 2.4. The condition that the density is less than 1 is equivalent to saying that there is a positive constant δ so that $\sqrt{-1} \partial\bar{\partial}\phi_r \geq (1 + \delta) \Upsilon_r^W$ for $r \gg 1$.

Remark 2.5. The reason the weighted average ϕ_r is used (as opposed to ϕ) is to make the theorem look similar to its one-dimensional version, where the weight ϕ_r appears naturally in the proof of necessity of the density. (It arises there from the use of Jensen's Formula.) Thanks to Lemma 1.1 in [PV-2014] which is in turn taken from [BO-1995, L-2001], it turns out that if $\partial\bar{\partial}\phi$ is bounded above by a multiple of the Euclidean metric, we may replace ϕ_r by ϕ and get equivalent L^2 norms.

We are finally in a position to state our main theorem.

Theorem 2.6. *Let $\phi : \mathbb{C}^n \rightarrow \mathbb{R}$ be a C^2 -smooth plurisubharmonic function satisfying $0 \leq \sqrt{-1}\partial\bar{\partial}\phi_r \leq C\omega$ for some positive constant C , and let $W \subset \mathbb{C}^n$ be a possibly singular uniformly flat, complex hypersurface such that $W = T^{-1}(0)$ where $T = T_1 T_2 \dots T_k$ is a holomorphic function with $T_i^{-1}(0)$ being uniformly flat, smooth complex hypersurfaces such that $D_\phi^+(W) < 1$. Then any holomorphic function f on W such that $\int_W |f|^2 e^{-\phi} \omega_0^{n-1} < \infty$ can be extended to a holomorphic entire function F satisfying $\int_{\mathbb{C}^n} |F|^2 e^{-\phi} \omega_0^n \leq K \int_W |f|^2 e^{-\phi} \omega_0^{n-1}$ for some constant $K > 0$ independent of f but possibly dependent on C .*

Remark 2.7. The C^2 -smoothness condition of theorem 2.6 is actually not necessary. Indeed, there is a sequence of plurisubharmonic functions ϕ_ϵ decreasing pointwise to ϕ_r . There is a corresponding sequence of holomorphic functions F_ϵ extending f satisfying $\int_{\mathbb{C}^n} |F_\epsilon|^2 e^{-\phi_\epsilon} \omega_0^n \leq K \int_W |f|^2 e^{-\phi_\epsilon} \omega_0^{n-1} < K \int_W |f|^2 e^{-\phi} \omega_0^{n-1}$. Thus a subsequence $F_\epsilon e^{-\phi_\epsilon/2}$ goes weakly to a function G in L^2 . This means that a subsequence F_ϵ converges to some function F almost everywhere and hence weakly in L^2 . This means that F is holomorphic, extends f , and satisfies the desired estimates.

Remark 2.8. For the purposes of algebraic geometry it is important to let ϕ be singular with non-zero Lelong number. Our proof relies strongly on the aforementioned lemma in [BO-1995, L-2001] which in turn requires an upper bound on $\partial\bar{\partial}\phi_r$. Removing this upper bound seems to require fundamentally new ideas.

3. PROOF OF THE MAIN THEOREM

Let $W_i = T_i^{-1}(0)$ and $f_i = f|_{W_i}$. We extend f using an inductive procedure. Using theorem 1 in [PV-2014], and the fact that W_1 is a uniformly flat, smooth hypersurface with $D_\phi^+(W_1) < 1$, we may extend f_1 to F_1 satisfying $\int_{\mathbb{C}^n} |F_1|^2 e^{-\phi} \leq C \int_{W_1} |f_1|^2 e^{-\phi} < \infty$ (where $C > 0$ is a constant independent of f). Let us assume that f_1, \dots, f_i have been extended to an entire function F_i satisfying the L^2 estimates. Let $\tilde{f}_{i+1} = f - F_i|_W$. If we manage to extend \tilde{f}_{i+1} from $W_1 \cup W_2 \dots \cup W_{i+1}$ to \tilde{F}_{i+1} with good estimates, then $F_{i+1} = F_i + \tilde{F}_{i+1}$ will extend f_1, \dots, f_{i+1} .

Before we implement this strategy we need to prove a lemma regarding the norm of $F_i|_W$

Lemma 3.1. *For any holomorphic function F on \mathbb{C}^n satisfying $\int_{\mathbb{C}^n} |F|^2 e^{-\phi} \omega_0^n < \infty$ and any smooth, uniformly flat hypersurface $V = \mathcal{T}^{-1}(0)$*

$$(3.1) \quad \int_V |F|^2 e^{-\phi} \omega_0^{n-1} \leq C \int_{\mathbb{C}^n} |F|^2 e^{-\phi} \omega_0^n$$

for some positive constant C which is independent of F .

Proof. By uniform flatness [OSV-2006] there exists an open cover of \mathbb{C}^n by balls B_α of some positive radius lying in $[a, 2a]$ such that in $B_\alpha \cap V$, the hypersurface V is locally a graph $y = g_u(x)$ over a small disc in the tangent space of any point z in B_α such that $|g_u(x)| \leq C_a |x|^2$ where C_a is independent of z . Moreover every point in \mathbb{C}^n is contained in at most N balls and similar properties hold for concentric balls with the double the radius of B_α (which we denote by \bar{B}_α). Let ρ_α be a partition of unity subordinate to the open cover defined by the B_α .

Hence $\int_V |F|^2 e^{-\phi} \omega_0^{n-1} = \sum_\alpha \int_{V \cap B_\alpha} \rho_\alpha |F|^2 e^{-\phi} \omega_0^{n-1}$. If we manage to prove that $\int_{V \cap B_\alpha} |F|^2 e^{-\phi} \omega_0^{n-1} \leq \int_{\bar{B}_\alpha} |F|^2 e^{-\phi} \omega_0^{n-1}$ for all α , then we will be done.

Fix a B_α (whose radius is r_α). Using uniform flatness we may assume without loss of generality that $V \cap B_\alpha$ is actually $(z_1 = 0) \cap B_\alpha$. Fix a point $z = (0, z_2, z_3, \dots, z_n)$ in $V \cap B_\alpha$. Using the lemma from [BO-1995, L-2001] mentioned earlier and uniform flatness we may assume without loss of generality that there exists a holomorphic function H on B_α such that $e^{-\phi}(z) = |H(z)|^2$ and $\|e^{-\phi}/|H|^2\|_{C^0(B_\alpha)} \leq C$ where C is independent of the point z (but H can potentially depend on z). Thus

$$\begin{aligned} |F|^2 e^{-\phi}(0, z_2, \dots, z_n) &= |FH|^2(0, z_2, \dots, z_n) \leq C \int_{|z_1| < r_\alpha} |FH|^2(z_1, z_2, \dots, z_n) \frac{\sqrt{-1}}{2} dz_1 \wedge d\bar{z}_1 \\ &\leq C \int_{|z_1| < r_\alpha} |F|^2 e^{-\phi}(z_1, z_2, \dots, z_n) \frac{\sqrt{-1}}{2} dz_1 \wedge d\bar{z}_1 \end{aligned}$$

Integrating over all $(0, z_2, \dots, z_n)$ in $V \cap B_\alpha$ we see that

$$\int_{V \cap B_\alpha} |F|^2 e^{-\phi} \omega_0^{n-1} \leq \int_{\bar{B}_\alpha} |F|^2 e^{-\phi} \omega_0^{n-1}$$

As mentioned earlier this is enough to finish the proof. \square

Lemma 3.1 implies that $\int_W |F_i|^2 e^{-\phi} \omega_0^{n-1} = \sum_j \int_{W_j} |F_i|^2 e^{-\phi} \omega_0^{n-1} \leq C \int_{\mathbb{C}^n} |F_i|^2 e^{-\phi} \omega_0^n$.

To extend $\tilde{f}_{i+1} = f - F_i|_{W_i}$ we use the following L^2 -extension theorem from [V-2007]:

Theorem 3.2. *Let X be a Stein manifold with Kähler form ω , $Z \subset X$ a smooth hypersurface, $e^{-\eta}$ a singular Hermitian metric for the holomorphic line bundle associated to the smooth divisor Z , and T a holomorphic section of this line bundle such that $Z = \{T = 0\}$. Assume that $e^{-\eta}|_Z$ is still a singular Hermitian metric, and that*

$$\sup_X |T|^2 e^{-\eta} = 1.$$

Let $H \rightarrow X$ be a holomorphic line bundle with singular Hermitian metric $e^{-\kappa}$ whose curvature $\sqrt{-1}\partial\bar{\partial}\kappa$ is non-negative in the sense of currents. Suppose also that

$$\sqrt{-1}\partial\bar{\partial}\kappa + \text{Ricci}(\omega) \geq (1 + \delta) \sqrt{-1}\partial\bar{\partial}\eta$$

for some positive number δ . Then for each section $f \in H^0(Z, H)$ satisfying

$$\int_Z \frac{|f|^2 e^{-\kappa}}{|dT|^2 e^{-\eta}} \frac{\omega^{n-1}}{(n-1)!} < +\infty$$

there is a section $F \in H^0(X, H)$ such that

$$F|_Z = f \quad \text{and} \quad \int_X |F|^2 e^{-\kappa} \frac{\omega^n}{n!} \leq \frac{C}{\delta} \int_Z \frac{|f|^2 e^{-\kappa}}{|dT|^2 e^{-\eta}} \frac{\omega^{n-1}}{(n-1)!},$$

where the constant C is universal.

By lemma 4.15 of [PV-2014] we see that \tilde{f}_{i+1} is divisible by $S_i = T_1 T_2 \dots T_i$. The following lemma is crucial.

Lemma 3.3.

$$\int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} \leq C \int_{W_{i+1}} |\tilde{f}_{i+1}|^2 e^{-\phi} \omega_0^{n-1}$$

where $C > 0$ is a constant independent of \tilde{f}_{i+1} .

Proof. The integral may be written as

$$\int_{W_{i+1}} \frac{|\tilde{f}_{i+1}|^2 e^{-\phi}}{|S_i|^2 e^{-\int_{B(z,r)} \ln(|S_i|^2)} |dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_{i+1}|^2)}} \omega_0^{n-1}$$

where we recall that $S_i = T_1 \dots T_i$. The function $|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_{i+1}|^2)}$ is bounded below by virtue of uniform flatness (lemma 4.11 in [PV-2014]). The expression $|S_i|^2 e^{-\int_{B(z,r)} \ln(|S_i|^2)}$ may be written as

$$\prod_{u=1}^i \left(|T_u|^2 e^{-\int_{B(z,a)} \ln(|T_u|^2)} \right) e^{\int_{B(z,a)} \ln(|S_i|^2) - \int_{B(z,r)} \ln(|S_i|^2)}$$

where a is chosen to be so small that $W_u^{-1}(0) \forall u = 1, 2, \dots, i$ is (as earlier) a graph $y = g_u(x)$ over a small disc (of radius $> a$) in the tangent space of z such that $|g_u(x)| \leq C_a |x|^2$ where C_a is independent of z (once again by uniform flatness [OSV-2006]).

Since expressions of the form $|T|^2 e^{-\int_{B(z,a)} \ln(|T|^2) d\mu(\tilde{z})}$ are independent of the choice of defining function T , we may replace T_u in the expression above with $y(\tilde{z}) - g_u(x(\tilde{z}))$. (Notice that the \tilde{z} -coordinates may be changed to the (y, x) -coordinates by means of an affine isometry.) It is easy to see that $e^{-\int_{B(z,a)} \ln(|y - g_u(x)|^2)}$ is uniformly bounded below independent of z . So we infer that

$$\begin{aligned} & \int_{W_{i+1}} \frac{|\tilde{f}_{i+1}|^2 e^{-\phi}}{|S_i|^2 e^{-\int_{B(z,r)} \ln(|S_i|^2)} |dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_{i+1}|^2)}} \omega_0^{n-1} \\ (3.2) \quad & \leq C \int_{W_{i+1}} \frac{|\tilde{f}_{i+1}|^2 e^{-\phi}}{\prod_{u=1}^i (|y(z) - g_u(x(z))|^2) e^{\int_{B(z,a)} \ln(|S_i|^2) - \int_{B(z,r)} \ln(|S_i|^2)}} \omega_0^{n-1} \end{aligned}$$

The factor $e^{\int_{B(z,a)} \ln(|S_i|^2) - \int_{B(z,r)} \ln(|S_i|^2)}$ is seen to be uniformly bounded below using the proof of lemma 4.11 in [PV-2014]. So far we have

$$(3.3) \quad \int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} \leq C \int_{W_{i+1}} \frac{|\tilde{f}_{i+1}|^2 e^{-\phi}}{\prod_{u=1}^i (|y(z) - g_u(x(z))|^2)} \omega_0^{n-1}$$

At this juncture choose an open cover of W_{i+1} by balls B_α in \mathbb{C}^n of a sufficiently small radius $\frac{\epsilon_0}{2} \leq b_\alpha \leq (k+1)\epsilon_0$ where k is a constant. In fact, by virtue of uniform flatness one may choose these balls so that uniform paracompactness holds [PV-2014]. Extend this to an open cover of \mathbb{C}^n with the

same properties. Choose a partition of unity ρ_α subordinate to this open cover. Inequality 3.3 may be written as

$$(3.4) \quad \int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} \leq C \sum_{\alpha} \int_{W_{i+1} \cap B_{\alpha}} \rho_{\alpha} \frac{\left| \tilde{f}_{i+1} \right|^2 e^{-\phi}}{\prod_{u=1}^{u=i} (|y(z) - g_u(x(z))|^2)} \omega_0^{n-1}$$

Using lemma 4.16 of [PV-2014] we see that, at the cost of increasing the radius of the balls B_{α} by a factor to get new, bigger balls \hat{B}_{α} , we have

$$(3.5) \quad \int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} \leq C \sum_{\alpha} \int_{W_{i+1} \cap \hat{B}_{\alpha}} |\tilde{f}_{i+1}|^2 e^{-\phi} \omega_0^{n-1}$$

Since each point of \mathbb{C}^n is contained in a finite number of balls (let us say N),

$$(3.6) \quad \int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} \leq CN \int_{W_{i+1}} |\tilde{f}_{i+1}|^2 e^{-\phi} \omega_0^{n-1}$$

□

Proof of theorem 2.6: We now extend \tilde{f}_{i+1} from $Z_{i+1} = W_1 \cup W_2 \dots \cup W_{i+1}$ to an entire function \tilde{F}_{i+1} in \mathbb{C}^n using theorem 3.2 by choosing X to be \mathbb{C}^n , $Z = Z_{i+1}$, $\kappa = \phi_r$ (with the understanding that according to remark 2.5 the norms defined by ϕ_r and ϕ are equivalent), H to be trivial, and $\eta = \int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)$. The density condition ensures that the curvature condition in theorem 3.2 is satisfied. The only hypotheses that of theorem 3.2 that needs to be checked now is the finiteness of $\int_Z \frac{|f|^2 e^{-\kappa}}{|dT|^2 e^{-\eta}} \frac{\omega_0^{n-1}}{(n-1)!}$. Indeed in our case,

$$\int_Z \frac{|f|^2 e^{-\kappa}}{|dT|^2 e^{-\eta}} \omega_0^{n-1} = \sum_{j=1}^{i+1} \int_{W_j} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi_r}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1} = \int_{W_{i+1}} \frac{\left| \frac{\tilde{f}_{i+1}}{S_i} \right|^2 e^{-\phi_r}}{|dT_{i+1}|^2 e^{-\int_{B(z,r)} \ln(|T_1 \dots T_{i+1}|^2)}} \omega_0^{n-1}$$

which by lemma 3.3 and remark 2.5 is bounded above by $C \int_{W_{i+1}} |\tilde{f}_{i+1}|^2 e^{-\phi} \omega_0^{n-1} \leq C \int_W (|f|^2 + |F_i|^2) e^{-\phi} \omega_0^{n-1}$ for some constant $C > 0$.

By lemma 3.1 we may conclude that $\int_W |F_i|^2 e^{-\phi} \omega_0^{n-1} \leq C \int_{\mathbb{C}^n} |F_i|^2 e^{-\phi} \omega_0^n$ which is in turn less than (by the inductive hypothesis) $C \int_{\mathbb{C}^n} |f|^2 e^{-\phi} \omega_0^n$.

As mentioned earlier, the definition $F_{i+1} = F_i + \tilde{F}_{i+1}$ completes the inductive step. Hence we are done. □

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